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FORMULÆ FOR GRADUATING TAXATION

[THE following article appeared in the *ECONOMIC JOURNAL*, 1920, under the title "Mathematical Formulæ and the Royal Commission on the Income-tax." The formulæ which it is the purpose of the article to recommend, though described as "Mathematical," involve no higher mathematics than the operations mentioned in the preceding article (Y). There is here emphasised a caution which was alluded to in that article: that formulæ are only means to ends which must be prescribed by other than mathematical considerations (Y, p. 258). Where and how much to lighten or tighten the pressure of taxation must be, as Professor Pigou teaches (below, p. 262), "arrived at by general reasoning." There is no connection between graduation undertaken for the purposes herein described and the schemes of the graduation-crank who assumes that a distribution of fiscal burdens must be equitable because it conforms to a neat and pretty formula.

It may be well perhaps to quote some evidence bearing on this point given by the present writer before the late Royal Commission on the Income-tax [Cmd. 288, 4]. Sir Josiah Stamp having asked with reference to curves representing different principles of graduation: "Is there anything which tells us which is really the more just curve?" (Q. 11,815), the answer was: "No, I am rather doubtful about the point at which you should lighten or tighten the tax. Certainly, from mere knowledge of curves, you cannot get any such ethical proposition as that. It must be from people who have knowledge of the facts, and make mathematics their servant to carry out their ideal. How they would get it, I am sure I don't know. . . ." But "some formulæ would assist common-sense." . . . (11,816). "It may assist you to have only one or perhaps two screws to turn; you can let out here and tighten there with more facility than you otherwise could; but as to showing exactly where you should tighten or not, that is beyond my science."]

The grounds on which the Commission reject the use of graduation formulæ are to be examined here. Their objections are summarised in the form of quotations from two expert witnesses (Report, 132).

I. The first passage, from the evidence of Dr. Stamp, is directed against the use of a formula purporting to express the ideal relation between amount of income and amount of tax. In the context Dr. Stamp shows that it would be very difficult to obtain "a single comprehensive tax return for a year," as the practical application of such a graduation form would require (9583).¹ Moreover, whereas different formulæ are proposed, "it is impossible to say which of the various curves truly represent that principle of equality of sacrifice they purport to embody" (9609). "That function is necessarily unknown," as Professor Pigou puts it (4274). Again, as Professor Pigou suggests, it is not enough to secure equal sacrifice; "minimum aggregate sacrifice" must be taken into account (4256). Where the data are so vague the deduction must be "in the air" (Pigou, 4271, 2).

Yet the premises, however inadequate to the deduction of a definite formula, may suffice for a certain negative conclusion. The ground which will not serve as the foundation of the elaborate edifice designed may yet be solid enough to support a battering-ram capable of being directed against simpler edifices in the neighbourhood. First (α), so far as equal sacrifice is supplanted by minimum aggregate sacrifice, whatever presumption in favour of progressive taxation is afforded by the principle of equal sacrifice becomes strengthened. "The case for (progressive) graduation is stronger" (Pigou, 4258). Now (β) some presumption in favour of progressive taxation is afforded by the principle of equal sacrifice. "The function," according to which the satisfaction attending income—and the sacrifice attending taxation—varies, is, indeed, "necessarily unknown" (4274). But something about that function is known, or at least strongly presumed, namely, that satisfaction as dependent on income increases at a rate which diminishes more rapidly than does the rate of increase pertaining to the simple function proposed by Bernoulli as apt to represent the relation of means to satisfaction.² At least it may be safely assumed that the function has not the opposite

¹ The bracketed figures refer to the paragraphs in the Evidence.

² In short it is presumed that the function is of the hyper-Bernoullian kind described above, §, p. 108; where evidence in favour of the presumption is submitted. More recently Professor Pigou in *Wealth and Welfare* lends his authority to the presumption; and it is now commonly, though not universally, accepted.

character, that which would justify regressive taxation.¹ Thus, if either of the propositions α and β —and, *a fortiori*, if both—hold good, it follows that the common arrangement according to which the rate of taxation (the ratio of the tax to the total, or to the taxable, income) rises to a certain maximum, and thereafter remains constant² (however large the income), is contrary to the (distributional) first principles of taxation. The graduation proposed by the Commission is open to this criticism, in so far as the rate of taxation, though not perfectly stationary, increases very gently for incomes above £20,000. It is not significant for the purpose of this negative conclusion that super-tax and the income-tax proper are based on returns for different years. The additions by which the “effective rate” for the “total” of income-tax and super-tax is computed by the Commissioners in the tables of the second Appendix to the Report are accurate enough to justify this criticism.³ Of course, the unprogressive character of the scheme may admit of justification on *productional* grounds. (*Cp.* 4012, 4119, 4356, 11787, *et passim.*)

II. The above use of the materials which some enthusiasts have attempted to use for the purpose of constructing an ideally just graduation is quite consistent with the view that such a construction is impossible. Having exposed that impossibility, Professor Pigou, in the course of his searching questions, goes on to educe “the real purpose of the thing” (4271). “When you have got certain points on your scale arrived at by general reasoning,⁴ then a mathematical formula can give you a means of interpolating.” The “interpolation” contemplated (by the present writer at least) is of the kind which, given a set of figures (forming, say, the “argument,” or first column of a table), deduces a corresponding set of figures (to form the entries in the table) by means of a *formula*, or definite function. It is thus that actuaries deduce the mortality at different ages by means of the Gompertz or Gompertz-Makeham Law. The Pearsonian curves which play a great part in modern statistics are of this type.

To interpolation of this kind it may be objected that, in the words of the second passage adduced by the Commission, “it would try the temper of all taxpayers” (Report 132). Or, as

¹ Above, §, p. 108.

² *Cp.* above, p. 240.

³ Nor need the objection that the income-tax does not form the *whole* of taxation (*cp.* above, p. 248) give us pause, with reference to so rough an estimate; especially when it is observed that indirect taxation (and part at least of local taxation) is proportional to expended income.

⁴ *Cp.* Stamp, “the common sense and instinctive judgment of the people” (Report, *loc. cit.*); Hopkins, “a matter of common judgment” (213).

Mr. Hopkins, the witness cited, says elsewhere, "a mathematical formula would try the intelligence of a large proportion of the general public." "The time of the tax officials would thus be occupied . . . in striving to appease distracted taxpayers by explaining the complications of the system" (4017). But why, it may be asked, should the general public want to have the complications explained? They are concerned only with the amount that each has to pay. A statement in the form of a table of the amounts calculated by the formula is sufficient (*cp.* 4337, 11,827). Cannot the general public read the dial of a town clock without going behind to inspect the works? Must not even the more intelligent be content with a general knowledge of the principle on which time is measured, without going into the niceties of "escapements," "gridiron pendulums," and the like? May not the general public be satisfied that the dictates of "general reasoning," "common-sense and instinctive judgment,"¹ are fulfilled by the scale of graduation, without comprehending the particular method of interpolation? This is one of the questions on which the logic of a student is of little weight against the judgment of official experts; so long, at least, as that judgment is confined to particular concrete cases—for instance, the testimony of Mr. Hopkins that mathematical formulæ have "no practical application to the income-tax as it stands in this country at the present day" (4079).

A similar remark applies to the objection described by the Commission as "most serious and almost insuperable," namely, "the necessity of determining the *exact* total income of the taxpayer" (Report 133).² But that this objection is not universally applicable may be shown by an *argumentum ad hominem* or *ad vigintiviros*, the Commissioners, whose own scheme is nearly as open to this objection as one founded on a mathematical formula. To show this let us compare their scheme with formulæ of graduation proposed in the *ECONOMIC JOURNAL*.³ We may employ the tests which they prescribe, namely, "(a) practicability, especially so far as the conditions under which total income may be calculated are concerned; (b) equity, that is to say, the necessity for increasing the rate of tax steadily as the total income

¹ Above p. 262, note 4.

² *Cp.* Hopkins (4017). "If the present simple methods of graduation were discarded in favour of some mathematical formula under which the rate of tax varied with, say, each pound of income, it would be necessary to know the *exact* amount of the total income before an assessment upon any part of the income could be accurately made."

³ Above, p. 249 *et seq.*

increases . . . ; (c) simplicity in explanation and ease in comprehension" (Report, 135). The comparison is most conveniently made in the simple case of the taxpayer being a "single" person, and the income all "investment." The taxation proposed by the Commission is twofold, income-tax (proper) and super-tax.

The first tax is arrived at thus. Deduct £135 from the total income, and on the remainder, the "taxable income," impose a tax of 3s. in the pound, up to the limit of £360 total income, that is, on £225 taxable income. On the portion of taxable income above that limit impose a tax of 6s. in the pound. This scheme may challenge comparison with one proposed in the *ECONOMIC JOURNAL*, according to which the rate of taxation is given by the formula

$$r \frac{x}{x + L};$$

where x is the taxable income, the remainder over and above an exempted minimum; r and L ¹ are constants to be adjusted to conditions such as those stated by Professor Pigou in a passage above cited.

With reference to the first tract of income from £135 to £360, it may be admitted that, at least in this country at the present day, the Commission's scheme passes the first test better than its rival. If the taxpayer is charged at the uniform rate throughout that tract, he is under less temptation to transfer his income by misrepresentation to a figure at which the rate is lower; and there will be less need of revising local assessments (Report, 133). But this advantage ends at the limit £360. Thereafter the "effective rate of taxation" on the total income, or on the taxable income—an equally appropriate conception—varies continuously for the Commission's scheme, just as well as according to the formula. If L is the limit at which the tax ceases to be simply proportional, and r is the standard rate (Report, 149), the

¹ This constant L , called M in article V, is not to be identified with the constant, also called M , there employed in Prof. Cassel's scheme to represent the maximum abatement. L might be regarded as the excess of the maximum abatement above the minimum of subsistence, say £135. But that conception is not sufficiently general. Thus the formula might well be adapted to present a set of rates corresponding to the scale (income-tax *plus* super-tax) proposed by the Commission between incomes of £2000 and £20,000. If we determine L (and r) so that the taxes at those points should be the same as given by the formula and by the Commission (for single persons and income all investment), L proves to be above £1500, the total abatement above £1600! In the words of the Commission, a "difficulty would be experienced in convincing people with small incomes" that taxpayers with large incomes might properly receive an abatement so much larger than that allowed to small incomes (Report, 138). See above, p. 248.

rate of taxation on the taxable income is $r \frac{x - \frac{1}{2}l}{x}$; where, as before, x is the taxable income.¹ This rate varies continuously with x just as much as that above given by the formula. Whatever practical expedients are employed to meet the difficulty in the one case are surely equally admissible in the other. On the first count, then, (a) the two candidates are equal. Considering next the third test (c), simplicity, can it be maintained that there is much difference in this respect between the two expressions for the effective rate on the taxable income, viz. $r \frac{x - \frac{1}{2}l}{x}$ and $r \frac{x}{x + L}$? Lastly, (b) as to equity and steady increase, let us compare the graduation determined by the formula with that prescribed by the Commission (for incomes above £360). Whereas there are two constants at our disposal, r and L , let us determine them so that the formula may coincide with the Commission's scheme at two points considered as the "certain points on your scale arrived at by general reasoning" (Pigou) or "instinctive judgment" (Stamp).² The limit £360 (£225 above the exempted minimum (£135)) may properly be taken as one of those points.³ Let us take £2000 for the other point.⁴ We have then two simple equations to determine r and L .⁵ The resulting values are $r = 0.32058$ (6s. 6d. in the pound); $L = 255.869$.⁶ Using these values for the constants, let us determine the tax according to the formula at several points, and compare the figures with those of the Commission.

TABLE I.—Comparing Taxes according to A the Scheme of the Commission, and B the Formula.

Income in pounds	360	500	700	1000	1500	2000
Tax A	33.75	75.75	135.75	275.75	375.75	525.75
Tax B	33.75	68.79	124.75	214.00	367.30	525.75

There seems to be no significant difference between the two

¹ The tax is $\frac{1}{2}rl + r(x - l) = r(x - \frac{1}{2}l)$. Accordingly the rate of taxation is $r \frac{x - \frac{1}{2}l}{x}$.

² Above, p. 262.

³ Especially if it be conceded that the formula is not to apply below that limit, the effective rate for the tract below (and at the limit) being the same as what it is in the Commission's scheme, viz. 3s. per pound on the taxable income.

⁴ Of course, if preferred, we might take for the other datum the "standard" ratio $r = .3$; with the result of greater cunsilience for the higher incomes, less for the lower ones.

⁵ (1) $r \frac{225^2}{L + 225} = 33.75$ (2) $r \frac{1865^2}{L + 1865} = 525.75$

The figures on the right of the equations are obtained from the directions given in Part I. of the Report; supplemented by Table I. in Appendix II.

⁶ Of course in practice round numbers would be employed.

sets of figures. A similar indifference is shown by a comparison of the effective rates of taxation.

TABLE II.—*Comparison of Effective Rates according to A the Commission Scheme, B the Formula.*

Income in pounds	360	500	700	1000	1500	2000
A	$1/10\frac{1}{2}$	$3/0\frac{1}{2}$	$3/10\frac{1}{2}$	$4/6$	$5/0$	$5/3$
B	$1/10\frac{1}{2}$	$2/9$	$3/7$	$4/3\frac{1}{2}$	$4/11$	$5/3$

The effective rates of taxation on the *taxable income* present a similar comparison. In point of equity and gradual increase there is nothing to choose between the two schemes.

The above comparison illustrates a property of some importance belonging to the sort of "interpolation" with which we are here concerned. If the constants are determined so that the formula shall fit exactly at a few points, it will generally be found to fit approximately in the neighbourhood of, and even to a considerable distance from, these points. The property was brought prominently under the notice of the Statistical Society on the occasion when Dr. Brownlee read a paper advocating a new formula for the graduation of a Mortality Table.¹

Now let us go on to the second tax with which the Commission deals, the super-tax. And let no difficulty be created by the circumstance that the two taxes are not based on returns for the same year. The task of interpolation is not hampered by that circumstance; test (a) is passed triumphantly. For all that we have to do is to find a simple formula which will adequately represent the scheme proposed by the Commission (Report, 152). Here is the scheme:

TABLE III.—*Showing Rates of Super-tax proposed by the Commission.*

Income in thousands	2	2.5	3	4	5	6	7	8	9	10	20
Rates in successive zone	$1/6$	2	$2/6$	3	$3/6$	4	$4/6$	5	5	5	

This scheme is like the "Scene of Man" according to Pope—"A mighty maze, but not without a plan." Over a portion of the tract dealt with, from three to nine thousand, a certain law is discernible. With reference to that portion we may say, in

¹ *Journal*, Vol. LXXXII. (1919). Dr. Major Greenwood, giving his own and Mr. Yule's experience, said "they could fit the same data to this (the same) degree of accuracy by formulae from two contradictory theories, the mathematical expressions of which were totally distinct one from another" (*loc. cit.* p. 67). Similar testimony was borne by another speaker (p. 75), with the reservation that the functions employed should be of a kind suited to the subject matter.

the words of a Commissioner, "the regularity of the curve ought to be sufficient almost to satisfy the soul of a mathematician" (4128). The satisfaction would be greater if, instead of making five steps each of length corresponding to £1000 and of height 6*d.*, there was a *continuous* rise in the rate; represented by a simple equation.¹ The total tax (above the point where regularity set in) would then be represented by a simple parabola instead of a discontinuous series of right lines.

But we are not concerned now with this partial regularity; our task is, rather, to represent the whole scheme by a simple formula. To carry in mind and comprehend the Commission's scheme, a number of features must be attended to; there are a great many "things to remember," in the phrase of one of the expert witnesses.² There is first the starting-point, £2000; but this need not be counted against the scheme, as any rival scheme must also have a starting-point. Then there are the length £500 and the height 1*s.* 6*d.* of the first step, two "things to remember"; likewise the length of the second step £500 and the height 6*d.*, two more things. Then we enter on a tract characterised by steps of equal length £1000, and equal height 6*d.* There are thus only two things to remember about that tract up to the point at which it stops, £9000, which makes another thing. The length and breadth of the next step (up to £10,000) count for two more things; and the step to £20,000 means two more.³ Altogether, in order to comprehend the scheme, *eleven* details must be carried in mind. A tyro at golf complained that he could only make a good stroke when he kept in mind, and simultaneously attended to, a dozen rules (as to the position and movement of his clubs and limbs) which he had been taught by the professional. Now suppose, as some experts teach, that all the points which the golfer should observe are summed up in this one commandment, "Keep your eye on the ball;" would not this be favourable to "simplicity in explanation and ease in comprehension" (test *c*)? That is the sort of advantage which an interpolating formula offers to the practice of finance. There can be found an expression for the super-tax at any point of the scale from £2000 to £20,000 which involves only *two* constants. Nor are they involved in a complicated fashion. Indeed, the expression is

¹ E.g., $y = rx$; x being the income above £3000, and y the rate on that excess. The total tax would be given by the *integral* of y , viz. $\frac{1}{2} rx^2$. Contrast the expression for the total tax according to the Commission's scheme given below. Note to p. 268.

² Mr. Hopkins, deprecating the use of a table giving the tax payable on each of several incomes.

³ *Cp.* below, p. 268.

simpler than the expression for the super-tax at a point in that part of the Commission scale which alone admits of a general expression, the tract beginning at £3000 and ending at £9000.¹ The rule to be proposed is nearly as simple as that which applies to the *first* tract of the Commission's *income-tax*; which is the simplest possible, simple proportion. According to that rule, if x is the taxable income, and t is the tax to be paid thereon, $t = rx$, where r is a proper fraction. It comes to the same to say that, if y is the disposable income, that which remains over after the tax t has been paid, $y = bx$, where $b = 1 - r$ (also a proper fraction). That is the formula for unprogressive taxation. For progressive taxation, let us put a formula which seems next to that in simplicity, namely:

$$\text{Log } y = b \text{ Log } x.$$

Mr. Hopkins, who so ably voices official objection to mathematical complications, would, it may be hoped, not object to the use of logarithms as calculated to try the temper and intelligence of the general public. For Mr. Hopkins himself employs logarithms for the purpose of exhibiting schemes of taxation. He writes: "The divisions of the line representing the amount of income in this and the following graphs are based not on the actual amount of the income, but on the logarithm of the amount of the income" (Royal Commission on Income-tax, Instalment II., p. 81 *et seq.*). Our graph would differ from his only in having *both* axes based on logarithms. Let the axis of x represent the logarithm of taxable income; and let the disposable income be measured on the axis of y . Then the simple equation, $y = bx$, represented by a line on a diagram, represents the (logarithm of the) disposable income corresponding to any assigned value of the (logarithm of the) taxable income. If another constant is required, there should be added to the expression for y another constant, say, A , or $\text{Log } a$.² With reference to the case under consideration, the ratio of any y to the corresponding x is taken to be about 0.9; and the

¹ If x is the amount in pounds of income above £3000 (and below £9000), and n is the number of *integer* thousands in x , the general expression for the super-tax on $3000 + x$ is according to the rules of the Commission:

$$\begin{aligned} & 87.5 + x \left(\frac{1}{10} + (n+1) \frac{1}{40} \right) - \frac{n(n+1)}{2} \frac{1}{40} 1000. \\ & \left[500 \frac{1.5}{20} + 500 \frac{1}{10} + 1000 \left(\frac{1}{10} + \frac{1}{40} \right) + 1000 \left(\frac{1}{10} + \frac{1}{40} \right) + \dots + 1000 \left(\frac{1}{10} + \frac{n}{40} \right) \right. \\ & \quad \left. + (x - n1000) \left(\frac{1}{10} + (n+1) \frac{1}{40} \right) = 87.5 \right. \\ & \quad \left. + x \left(\frac{1}{10} + (n+1) \frac{1}{40} \right) - n(n+1) \frac{1}{40} 1000 + \frac{n(n+1)}{2} \frac{1}{40} 1000, \right] \end{aligned}$$

² The equation $\text{Log } y = \text{Log } a + b \text{ Log } x$ is identical with the perhaps less familiar form given above, $y = ax^b$.

constant addendum 0·32; constants determined as follows. To fit the formula to the Commission's super-tax, assumed to be agreeable to the judgment of the wise, let us take as the points "arrived at by general reasoning," in Professor Pigou's phrase, £2000 and £10,000; for which the amounts of super-tax are respectively *nil* and £1462 (for single persons, with income all "investment"; Report, Appendix II., Table No. 4). We have thus two simple equations¹ whereby to determine the constants *a* and *b*. Solving these equations, we obtain: $\text{Log } a = 0\cdot32419$, $b = 0\cdot90179$. These constants are employed to find the disposable income and thence the tax payable for any amount of income between £2000 and £20,000. Some comparisons with the scheme of the Commission are presented.

TABLE IV.—*Comparing Super-tax A according to the Commission, and B according to the Formula.*

Income in pounds	2000	5000	7000	10,000	15,000	20,000
Super-tax A	0	362	737	1462	2712	3962
Super-tax B	0	443·4	853·6	1462	2697	4050 ²

The comparison of these figures with those proposed by the Commission will show that test (*b*) is satisfied by the new curve.

No doubt, if we advance far beyond £20,000, the characteristic property of interpolation which has been noticed³ will cease to operate; there arises discrepancy of the kind shown below:

TABLE V.—*Comparing the Effective Rates of Super-tax on Incomes above £20,000 according to A the Commission's Scheme, and B the Formula.*⁴

Income in pounds	20,000	50,000	100,000	500,000	1,000,000
Effective rate A	$3/11\frac{1}{2}$	$4/10\frac{1}{2}$	$5/2\frac{1}{2}$	$5/5$	$5/5\frac{1}{2}$
Effective rate B	$4/0\frac{1}{2}$	$5/6$	$6/4\frac{1}{2}$	$8/4\frac{1}{2}$	$9/1\frac{1}{2}$

¹ (1) $\text{Log } a + b \text{ Log } 2000 = \text{Log } 2000$; (2) $\text{Log } a + b \text{ Log } 10,000 = \text{Log } (10,000 - 1462)$.

² Of course other *pivots* might have been used for the purpose of obtaining a formula of the proposed type corresponding to the scheme of the Commission (presumed to be just); for instance, 2000 as before, and 7000 for which the super-tax, according to the Commission, is 737. Whence by parity of reasoning there is found $b = \cdot91101$ and $\text{Log } a = \cdot29376$. Which constants being inserted in the formula will give a scale similar in its general features to Table IV.

³ Above.

⁴ When an income-tax of so much (*e.g.*, 6s.) per pound is added to a super-tax framed as above, there will *ultimately* (for incomes above some millions sterling per ann.!) be reached a stage at which the tax-payer will have no motive to increase his income (*see* above, p. 251 *et seq.*). But, if necessary, this breakdown can be indefinitely deferred by putting for "Log *x*" in the formula " $\text{Log } (1 - \gamma)x$;" where γ is a proper fraction not exceeding the rate of the income-tax (*e.g.*, $\cdot3$).

If, then, the flattening of the rate for the upper incomes was deliberate on the part of the Commission, the new formula will cease to correspond to their judgment. But if there was a minority, as we may surmise,¹ in favour of a more severe progression, they might welcome a formula which embodies their ideal without any additional complication; whereas, to effect a similar graduation (above £20,000) on the lines of the Commission, some dozen additional "things to remember" might be required. Or, if the requirements of a just progression were thought not to be quite satisfied by the formula, it would be easy to introduce one or more additional functions of the same type, tightening or lightening the tax at different points according to the judgment of experts and the sense of the community.² The Chancellor of the Exchequer, employing a formula which of all appropriate to progressive taxation seems one of the simplest,³ may hope with some confidence to construct a scale of taxation not very different from the ideal scale, could it be discovered. He would be in the fortunate position of the statistician when, having to combine observations dispersed according to some unknown law, he selects, out of the innumerable possible methods of averaging, a familiar one—the arithmetic mean; or when, in the construction of an index-number, he employs some handy system of weights, the true system not being ascertainable. The statistician may be certain that the methods practised will not yield the best result conceivable; but he may also presume that the result of the ideally best method is likely not to differ widely from that of his fairly good method. Something of the confidence which the theory of probabilities imparts to statistics the characteristic property of interpolation⁴ may impart to fiscal practice.

Altogether, the Commission seem not to have done justice to the use of mathematical formulæ for the purpose of interpolation. They justly reject formulæ purporting to express an ideal scale of taxation. But they do not recognise that some approach to the ideal is possible by way of interpolation.

¹ Asked if he agreed to graduation leading to "practical extinction beyond a certain point," Dr. Stamp replied, "I hesitate to say extinction, but I think severe progression" (1901).

² *Cp.* above, p. 262.

³ As the classical utility-function $\log x/c$ (where x is the total income and c the minimum of subsistence), perhaps the simplest function which fulfils the condition of continual increase at a decreasing rate, leads to the (unprogressive) rate of taxation $r = \text{constant}$; so the next simplest function that increases less rapidly, viz. $\log \log x/c$, leads to the (progressive) rate $r = \text{const.} \times \log \frac{x}{c}$.

⁴ Above, p. 266.